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STUDY OF THE POSSIBILITY OF SOLVING THE COUPLING EQUATION FOR THE SABININ-YURIEV MODEL IN AN ANALYTICAL FORM

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Abstract. The effectiveness of converting industrial sites (IS) of coal industry enterprises (CIE) into wind power plants is determined not only by the justification of the rotor diameter, the distribution of the blade angle of rotation and the chord length, but also by choosing a certain blade cross-sectional profile, the aerodynamic properties of which are preferable in these conditions. The results of the study of the dependencies of the aerodynamic characteristics of more than 50 most well-known airfoils on the trigonometric functions of the angle of attack allowed us to propose new approximation dependencies. These dependencies simplify the assessment of the prospects of using different types of profiles for wind turbine rotors (WTR) with a horizontal axis of rotation in the conditions of the IS of CIE. This allowed us to develop an analytical method for solving the coupling equation in the Sabinin-Yuriev mathematical model of the process of interaction of air flow with the rotor of a wind turbine with a horizontal axis of rotation. This equation relates the parameters of the wind flow, rotor rotation conditions, aerodynamic and geometric qualities of the rotor, and the values of inductive velocities introduced into the air flow by the rotor of a wind turbine with a horizontal axis of rotation. As a result of this improvement of the Sabinin-Yuriev model, an analytical solution to the coupling equation was obtained for the first time, which allows avoiding iterative algorithms in calculating the inductive velocities introduced into the air flow by the rotor of a wind turbine at given wind flow parameters, rotor rotation conditions, aerodynamic and geometric parameters of the rotor. The proposed formulas, for the first time, allow us to create algorithms for calculating the characteristics of WTR with a horizontal axis of rotation without the use of iterative procedures associated with the numerical solution of the coupling equation and simplify the solution of problems of optimizing the parameters of the WTR with a horizontal axis of rotation, thereby ensuring their maximum adaptation to the conditions of the IS CIE.

Keywords: wind turbine with a horizontal axis of rotation, wind energy utilization factor, Sabinin-Yuriev model, rotor speed, angle of attack.

1. Introduction

The threat of turning the regions of Ukraine where CIE facilities are located into depressed regions remains relevant and is of increasing interest to domestic and foreign scholars [1–3]. Most often, experts suggest re-profiling CIE facilities into enterprises that will produce electricity and heat. However, it is proposed to obtain these energies from different sources: through the utilization of coal mine methane gas, by processing coal preparation waste accumulated in storage facilities, by burning local fuels and pellets of own production, or from renewable energy sources. Recently, an integrated option has been increasingly proposed, when the CIE is focused on the use of several of the above options. At the same time, the use of wind energy is considered universal and can be used both during the operation of the CIE for its intended purpose and during its modernization, simultaneously with the processing of coal preparation waste and utilization of mine gas, as well as after the exhaustion of coal-related sources. Thus, one of the promising areas of CIE diversification is the creation of wind power plants on their ISs by locating wind turbine parks [1–6]. For the conditions of the eastern regions of Ukraine, where most of the CIE are located, low-speed wind turbines with a horizontal axis of rotation are considered the most promising. Such WTRs are designed to operate at low wind speeds, which allows them to operate for a significant part of the year and makes it possible to use standard aerodynamic profiles for the blades [7 – 9]. Today, about

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fifty profiles are known and well studied, and information on their manufacture and calculation of their aerodynamic qualities is publicly available [10–17]. That is, for the conditions of a particular IS of CIE, the highest efficiency of a wind turbine can be ensured not only by air speed, rotor diameter, distribution of the blade angle of rotation, and chord length, but also by choosing a certain sectional profile whose aerodynamic properties are preferable under these conditions [18–22]. The latter factor in increasing efficiency is limited by the amount of calculations to be performed and the unsuitability of existing mathematical models for such calculations. Today, the most common mathematical models of the process of interaction of the air flow with the rotor of a wind turbine with a horizontal axis of rotation are: a disk vortex model, a model of an ideal wind wheel, a model of elementary jets, and the Sabinin-Yuriev model.

However, all of these models, except for the last one, take into account the aerodynamic characteristics of the profiles to a very limited extent. The Sabinin-Yuriev model was developed to calculate the parameters and operating modes of helicopter rotorcraft and has been experimentally tested. Adaptation of this model to wind turbine rotors with a horizontal axis of rotation does not require significant effort. It is necessary to rebuild the velocity triangle in the diagram of interaction between the profile and the air flow, taking into account that this is not a bearing rotor, but a profile that is bumped by the air flow (Fig. 1) [7–9].

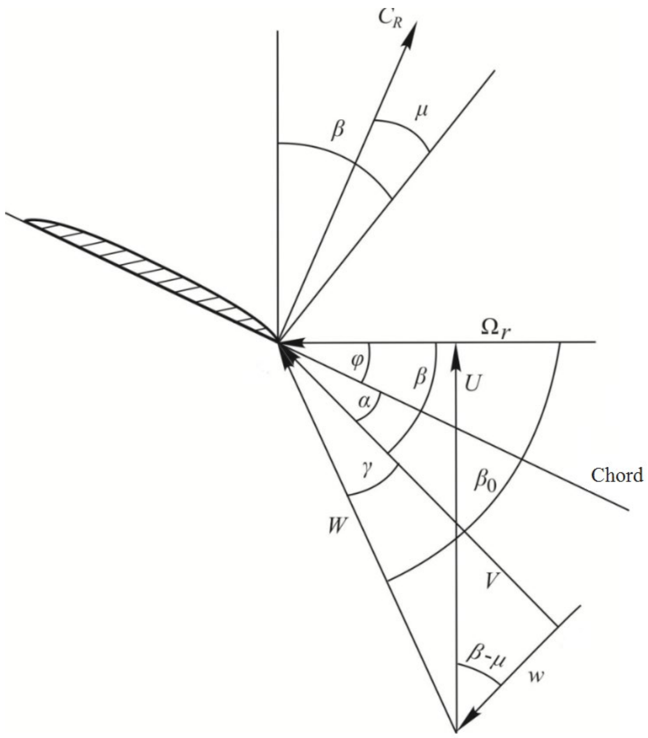

Figure 1 – Diagram of air velocity distribution and aerodynamic forces in the radial section of the wind turbine rotor blade with a horizontal axis of rotation

The following notations were adopted in the figure:
W – vector of the total velocity of the undisturbed flow; V – vector of the total

velocity of the disturbed flow; w – vector of the total induction velocity due to the presence of a rotor in the flow; β – angle formed between the direction of the vector of the total velocity of the disturbed flow and the plane of rotation of the rotor; α - angle of attack; φ – cross-sectional twist angle; Ω_r – rotor rotation frequency; r – rotor radius; U – air velocity along the axis; ,C_R – coefficient of total aerodynamic force; β_0 – flow angle, which is formed between the direction of the full velocity vector of the undisturbed flow and the rotor rotation plane; γ – bevel angle, i.e., the angle formed between the directions of the full velocity vectors of the undisturbed and disturbed flows; μ – angle of deviation of the total aerodynamic force from the normal to the profile chord.

In addition, when using the calculation formulas and solving the coupling equation, it should be remembered that the rotor speed is not a known quantity and its value cannot be changed arbitrarily. The results of the analysis of the calculated dependencies and equations of the Sabinin-Yuriev model indicate the following advantages: when determining the inductive velocities introduced into the air flow by the wind turbine rotor, both the lift coefficient and the drag coefficient are taken into account; it is possible to design rotors with blades whose chord length and angle of rotation vary along the radius; when determining the forces and moments acting on the WTR with a horizontal axis of rotation, the presence of terminal losses on the WTR blades is taken into account; it is taken into account that the presence of a rotor in the air flow leads not only to a decrease in the absolute value of the velocity vector, but also changes its direction; when determining the angle of attack, the influence of induced velocities is taken into account.

The disadvantages of the Sabinin-Yuriev model include the impossibility of analytically solving the coupling equation that links the wind flow parameters, rotor rotation conditions, aerodynamic and geometric qualities of the rotor, and inductive velocities introduced into the air flow by the wind turbine rotor. This equation must be solved for each blade section where the parameters are calculated, which greatly complicates the calculations and does not allow for the selection of rational parameters and the study of dependencies on extremes. Thus, the purpose of this publication is to propose an analytical method for solving the coupling equation in the Sabinin-Yuriev mathematical model of the process of interaction of air flow with the rotor of a wind turbine with a horizontal axis of rotation for discussion by the scientific community.

2. Methods

The final losses on the WTR blades in the Sabinin-Yuriev model are taken into account by excluding from the calculation the cross-sections of the blades that are below 0.1 and above 0.9 of the rotor radius. According to this theory, the power of WTR with a horizontal axis of rotation depends on the air flow rate, rotor radius and the efficiency of interaction between the air flow and the rotor [7 – 9]:

$$P = C_p \frac{\pi}{2} R^2 \rho U^3; \quad (1)$$

$$C_p = 8\lambda \int_{i_0}^{1} G\sqrt{1+\lambda^2 \bar{r}^2}\, \Psi \bar{r}^2 d\bar{r}, \quad \Psi = \frac{\cos^3(\gamma+\mu) - \cos^2(\gamma+\mu)\sin(\gamma+\mu)}{\cos^2\mu}; \quad (2)$$

$$\lambda = \frac{\Omega R}{U}; \quad G = \frac{C_R \sigma}{4}; \quad i_0 = \frac{R_1}{R}; \quad \sigma = \frac{NC}{2\pi R}; \quad \bar{r} = \frac{r}{R},$$

where P – wind turbine power; C_P – wind energy utilization factor [7–9]; R – rotor radius; ρ – air density; λ – rotor speed; r – radius of the cross-section location; $\bar{r}$ – relative radius of the current section; R_1 – rotor hub radius; i_0 – dimensionless rotor hub radius; G – Yuriev's constant [7–9]; Ω – rotor rotation frequency; N – number of blades in the rotor; σ – rotor solidity factor; C – length of the blade chord in cross-section; μ – angle of deviation of the total aerodynamic force from the normal to the profile chord.

The main relationship of the Sabinin-Yuriev model is the coupling equation that links wind flow parameters, rotor rotation conditions, aerodynamic and geometric qualities of the rotor, and inductive velocities introduced into the air flow by the wind turbine rotor:

$$\sin(\beta_0 - \gamma)\sin\gamma = G\cos(\gamma + \mu(\beta_0 - \gamma - \phi)); \; G = \frac{\sigma}{4} C_R(\beta_0 - \gamma - \phi), \quad (3)$$

where β_0 – the angle of flow incidence formed between the direction of the total velocity vector of the undisturbed flow and the plane of rotation of the rotor.

The coupling equation for the Sabinin-Yuriev model is based on the simultaneous consideration of the fundamental conservation laws and the velocity triangle in a separate section of the rotor blade. This coupling equation is expressed by the difference of two terms. The first of them is the product of two sines of the angle characterizing the inductive velocities and the difference of the angle formed between the direction of the total velocity vector of the undisturbed flow and the plane of rotation of the rotor and the angle characterizing the inductive velocities. The second term contains the product of the coefficient of the total aerodynamic force, the rotor filling coefficient and the cosine of the sum of the angle characterizing the inductive velocities and the angle by which the total aerodynamic force deviates from the perpendicular emanating from the leading edge of the profile. Analytical solutions to the equation are not known. Some researchers have tried to reduce this equation to a quadratic equation of the fourth degree, but this was possible only if the dependence of the coefficient of the total aerodynamic force on the angle characterizing the inductive speeds was ignored. But this approach requires additional justification, at least for those types of profiles that are used for wind turbine rotors with a horizontal axis of rotation.

The results of the study of the intervals of quantities included in the coupling equation in the Sabinin-Yuriev model indicate that the right and left parts of the equation are always greater than zero, but less than unity. The left part of the equation can be equal to zero only in two cases that are not its solutions: when the angle characterizing the inductive velocity is equal to zero, or it is equal to the angle formed between the direction of the total velocity vector of the undisturbed flow and the plane of rotation of the rotor. In the first case, the wind turbine rotor does not exist in the flow, because there is no flow disturbance that it introduces. In the second case, the rotor blades rotate with the maximum plane to the air flow and the moment that turns the rotor is absent.

The initial data when solving the coupling equation in the Sabinin-Yuriev model are: the angle formed between the direction of the total velocity vector of the undisturbed flow and the rotor rotation plane; the coefficient of the total aerodynamic force; the rotor filling factor. The angle characterizing the inductive velocity is the independent variable. The peculiarity of solving the coupling equation in the Sabinin-Yuriev model by numerous methods is that at the boundary of the interval of the existence of the root, the left part of the equation turns into zero. This makes it impossible to use the dichotomy method.

It is impossible to assess the stability of the ordinary iteration method by the theorem on compressible images, since the requirement of not exceeding the absolute value of the first derivative of unity, in the case of this equation, turns into an equally complex and nonlinear equation.

In view of the above, and taking into account that the functions in the right and left parts of equation (2) are represented by ordinary trigonometric functions, the most likely, in terms of obtaining an analytical solution, is to simplify the calculations by approximating the dependence of the aerodynamic characteristics of the profile not on the angle of attack, but on its trigonometric functions.

3. Theoretical part

Information was searched and summarized regarding the aerodynamic properties and types of blade profiles of wind turbine rotors with a horizontal axis of rotation. As a result of a search of publicly available sources, information was found on body profiles that are not S-shaped, such as NASA-00, NASA-45M, Su 26, P 52, Yak 55, 35 A and 35 B, NAVY N60, N-10 and N-18, GA(W)-1, V-16, MDC-27, MUNK, MHTC, Clark-Y, Clark-Y 15 and Clark-YH, MVA-123 and MVA-301, FX60, RAF, Getting, as well as TsAGI series A, B and BS, P-II and P-III, 6, 718 – 723, 730 – 734, 790, 796, 831, 846, 909, 911 and “ESPERO”, NACA series 44, 43 and 23, Go-535 and Go-549, DEF-P9-14, MS 16/209, Me-163, K-3 and S. Chaplygin Siberian Research Institute of Aeronautics and Astronautics. The results of the analysis of the aerodynamic characteristics of the existing profiles indicate a significant nonlinearity of the dependences of the coefficient of total aerodynamic force and angle μ on the angle of attack (Table 1–2). The presence of an extremum of these values in the middle of the interval poses significant difficulties in the calculation and complicates the approximation of the aerodynamic characteristics of the profiles.

Therefore, instead of using coefficient C_P and angle μ it is recommended to use the coefficients of lift and drag force, which are easily introduced into the right-hand side of equation (3) if the formula for the cosine of the sum of the angles is applied:

$$\frac{4}{\sigma}\sin(\beta_0-\gamma)\sin\gamma = C_y\cos\gamma - C_x\sin\gamma\,;\; C_y = C_R\cos\mu,\; C_x = C_R\sin\mu\,. \qquad (4)$$

Table 1 – Parameters of aerodynamic characteristics of some profiles at the beginning and at the end of the change interval

Profile	At the beginning of the interval			At the end of the interval		
	α, °	C_P	μ, °	α, °	C_P	μ, °
A-9%	0.0	0.080	5.86	16.0	1.090	7.70
A-12%	0.0	0.142	4.35	18.0	1.035	9.90
A-15%	0.0	0.184	3.80	18.0	0.998	10.87
A-18%	0.0	0.210	3.87	18.0	1.241	7.63
A-21%	0.0	0.249	4.15	20.0	1.353	8.04
B-8%	0.0	0.027	13.42	16.0	0.813	13.00
B-12%	0.0	0.063	6.79	16.0	0.957	6.09
B-16%	0.0	0.100	4.69	18.0	0.972	7.03
B-20%	0.0	0.134	4.06	16.0	0.965	6.01
P-II 10%	0.0	0.150	4.42	20.0	1.148	10.00
P-II 14%	0.0	0.219	4.10	18.0	1.445	6.36
P-II 16%	0.0	0.204	4.45	22.0	1.438	9.13
P-III 15.5%	0.0	0.300	0.34	24.0	1.780	0.81
Mynk-1	1.5	0.120	3.67	18.0	0.829	18.10
Mynk-2	1.5	0.097	5.13	15.0	0.911	7.45
Mynk-3	0.0	0.017	35.28	18.0	1.071	8.74
Mynk-6	0.0	0.016	2.86	21.0	1.184	9.05
Mynk-12	0.0	0.096	5.42	21.0	1.186	10.71
Mynk-15	0.0	0.227	3.25	21.0	1.196	11.91
Clark-YH-8%	0.0	0.082	5.02	14.0	0.985	5.08
USA-27	0.0	0.332	2.76	18.0	1.336	7.81
35A	0.0	0.421	4.09	24.0	1.508	11.90
35B	0.0	0.378	2.64	18.0	1.321	9.32
NAVY N60	0.8	0.425	1.34	14.6	1.617	1.75

Unlike the dependences of the coefficient of total aerodynamic force and angle on the angle of attack, the dependences of coefficients C_y and C_x on the angle of attack are not extreme and allow, after minor processing, approximation by linear functions of the sine or cosine of the angle of attack.

The results of the numerical studies that have been performed indicate that the dependence of the lift coefficient on the angle of attack with engineering accuracy can be approximated by the following function:

$$C_y = A_y + B_y\sin\alpha\,;\qquad A_y = C_y^{\min} + A\Delta C_y\,;\qquad B_y = B\Delta C_y\,, \qquad (5)$$

where $C_y^{\min}$ minimum value of the lift coefficient on the interval; A, B –

approximation parameters; ΔC_y – the maximum difference between the value of the lift coefficient on the interval and its minimum value.

Table 2 – Parameters of aerodynamic characteristics of profiles of TsAGI and NASA at the beginning and at the end of the change interval

Profile	At the beginning of the interval			At the end of the interval		
	α, °	C_P	μ, °	α, °	C_P	μ, °
TsAGI -6-8,2%	0.0	0.168	4.16	12.0	0.926	5.37
TsAGI -6-12%	0.0	0.171	4.58	20.0	1.186	9.86
TsAGI -6-13%	0.0	0.185	4.02	14.0	1.055	5.44
TsAGI -6-16%	0.0	0.215	4.06	20.0	1.262	9.20
TsAGI -6-19%	0.0	0.213	4.26	24.0	1.301	11.35
TsAGI -6-20%	0.0	0.177	5.52	20.0	1.266	8.45
TsAGI -719	0.0	0.317	4.24	22.0	1.231	14.79
TsAGI -731	0.0	0.138	3.15	16.0	1.109	7.39
TsAGI -732	2.0	0.298	2.27	16.0	1.016	12.60
TsAGI -734	0.0	0.078	6.15	16.0	0.994	9.51
TsAGI -831	0.0	0.309	3.42	20.0	1.057	17.66
NASA-0006	2.0	0.150	2.67	24.0	0.920	25.52
NASA-0009	4.0	0.300	2.67	26.0	0.991	23.32
NASA-0012	2.0	0.150	3.44	22.0	1.560	6.37
NASA-0015	2.0	0.150	3.44	20.0	1.427	5.63
NASA-0018	2.0	0.141	4.90	20.0	1.397	5.75
NASA-0021	2.0	0.150	4.58	20.0	1.387	5.80
NASA-2210	0.0	0.120	5.05	22.0	1.164	13.83
NASA-2212	0.0	0.122	5.15	18.0	1.179	7.21
NASA-2217	0.0	0.131	5.71	22.0	1.120	11.39

The results of the performed numerical studies indicate that the dependence of the drag coefficient on the angle of attack with engineering accuracy can be approximated by the following function:

$$C_x = A_x - B_x \cos\alpha, \qquad A_x = C_x^{\min} + B_x, \quad B_x = E\Delta C_x, \tag{6}$$

where $C_x^{\min}$ – minimum value of the drag coefficient on the interval; E – approximation parameter (Table. 3); ΔC_x – maximum difference between the value of the drag coefficient on the interval and its minimum value.

Dependencies (5) and (6), unlike the known ones, establish the dependence of the coefficients of aerodynamic forces not on the angle of attack, but on the trigonometric functions of this angle, which allows us to take into account in the coupling equation the dependence of these parameters on the flow angle, which is determined by inductive velocities.

The restrictions associated with the dependence of the angle μ on the angle γ are removed by using formulas (5) and (6), which allows us to rewrite the coupling equation (4) in the following form:

$$A_y cos\gamma - A_x \sin\gamma + W = 0\,; \tag{7}$$

$$W = B_y \sin(\beta_0 - \phi - \gamma)\cos\gamma + B_x \cos(\beta_0 - \phi - \gamma)\sin\gamma - \frac{4}{\sigma}\sin(\beta_0 - \gamma)\sin\gamma\,. \tag{8}$$

Table 3 – Parameters of approximation of aerodynamic characteristics of profiles

Profile	*A*	*B*	*E*	Profile	*A*	*B*	*E*
A-9%	0.0438	3.6307	23.77	TsAGI-734	0.0218	4.2216	22.33
A-12%	0.0165	4.3171	19.26	TsAGI-831	0.0498	3.2286	14.14
A-15%	0.0235	4.4318	19.26	Mynk-1	0.0401	3.6860	21.62
A-18%	0.0300	3.5135	19.51	Mynk-2	-0.0843	4.5720	28.33
A-21%	0.0336	3.1748	16.11	Mynk-3	0.0091	3.8914	18.66
B-8%	0.0752	3.8882	24.81	Mynk-6	0.0121	3.4464	13.75
B-12%	0.0252	3.7738	25.80	Mynk-12	0.0137	3.3190	13.60
B-16%	0.0480	3.4368	20.41	Mynk-15	0.0489	3.4824	14.20
B-20%	0.0233	3.7700	25.85	NASA-0006	0.0235	3.927	12.79
P-II 10%	0.0209	3.4206	16.05	NASA-0009	-0.2922	4.2528	10.17
P-II 14%	0.0028	3.2972	20.29	NASA-0012	-0.0999	3.0287	13.66
P-II 16%	0.0191	3.1022	13.38	NASA-0015	-0.1067	3.2763	16.93
P-III 15.5%	0.0094	2.4684	11.42	NASA-0018	-0.1025	3.2806	16.94
TsAGI -6-8.2%	0.0044	4.8479	44.27	NASA-0021	-0.1098	3.2395	16.20
TsAGI -6-12%	0.0309	3.3882	15.60	NASA-2210	0.0091	3.7827	12.67
TsAGI -6-13%	0.0475	4.2446	33.80	NASA-2212	0.0034	3.6866	19.34
TsAGI -6-16%	0.0533	3.1040	16.06	NASA-2217	0.0252	3.4389	13.48
TsAGI -6-19%	0.0787	2.5642	11.21	Clark-YH-8%	0.0071	4.1441	33.61
TsAGI -6-20%	0.0408	3.0137	16.06	USA-27	0.0173	3.6345	18.59
TsAGI -719	0.0455	3.4361	12.88	35A	0.03	3.0322	11.36
TsAGI -731	0.0528	3.7690	23.79	35B	0.0176	3.9585	19.02
TsAGI -732	-0.0712	4.3859	24.67	NAVY N60	-0.0358	4.1544	26.03

As can be seen from formulas (7) and (8), this form of the coupling equation in its present form expresses the dependence of the flow bevel angle on the characteristics of the blades and the conditions of air flow onto the rotor area. Using the formulas for the product of trigonometric functions, formula (8) can be rewritten as follows

$$W = \frac{B_y + B_x}{2}\cos\phi\sin(\beta_0 - 2\gamma) - \left[\frac{B_y + B_x}{2}\sin\phi + \frac{2}{\sigma}\right]\cos(\beta_0 - 2\gamma) + A_0\,; \tag{9}$$

$$A_0 = \frac{B_y + B_x}{2}\sin(\beta_0 - \phi) + \frac{2}{\sigma}\cos\beta_0\,.$$

Applying the known formulas for transforming the sums of trigonometric functions in equations (7) and (9) to sines and cosines, we obtain the relationship equation in one of the following forms:

$$A_y \cos(\theta' + \gamma) - A_{2y} \cos(\beta_0 + \Theta' - 2\gamma) + A_0 = 0; \tag{10}$$

$$A_y \sin(\theta - \gamma) + A_{2y} \sin(\Theta - \beta_0 + 2\gamma) + A_0 = 0; \tag{11}$$

$$A_y = \sqrt{A_y^2 + A_x^2}\,;\, A_{2y} = \frac{1}{2}\sqrt{(B_y + B_x)^2 + \frac{8}{\sigma}(B_y + B_x)\sin\phi + \frac{16}{\sigma^2}}\,;\ \theta' = arctg\left(\frac{A_y}{A_x}\right); \tag{12}$$

$$\theta = arctg\left(\frac{A_x}{A_y}\right);\ \Theta = arctg\left[\frac{(B_y + B_x)\sin\phi + \frac{4}{\sigma}}{(B_y + B_x)\cos\phi}\right];\ \Theta' = arctg\left[\frac{(B_y + B_x)\cos\phi}{(B_y + B_x)\sin\phi + \frac{4}{\sigma}}\right].$$

Given the intervals of angle change γ, the trigonometric functions in equations (10) and (11) can be expanded into power series and the first two terms can be preserved:

$$\sin \approx x - \frac{x^3}{6}; \qquad \cos \approx 1 - \frac{x^2}{2}. \tag{13}$$

Applying (13) to the terms of equation (11), after appropriate transformations, we obtain the following equation for determining the angle γ:

$$\gamma^3 - p_2\gamma^2 + p_1\gamma + p_0 = 0;$$

$$p_2 = 3\frac{A_\gamma\theta - 4(\Theta - \beta_0)A_{2\gamma}}{A_\gamma - 8A_{2\gamma}};\ p_1 = 3\frac{A_{2\gamma}\left(4 - (\Theta - \beta_0)^2\right) - \left(2 - \theta^2\right)A_\gamma}{A_\gamma - 8A_{2\gamma}};$$

$$p_0 = \frac{6A_0 + A_\gamma\left(6 - \theta^2\right)\theta + A_{2\gamma}(\Theta - \beta_0)\left(6 - (\Theta - \beta_0)^2\right)}{A_\gamma - 8A_{2\gamma}},$$

which can be solved using Cardano's formulas.

Applying (13) to the terms of equation (10), after appropriate transformations, we obtain the following formula for determining the angle γ:

$$\gamma = \begin{cases} \Lambda\left(1 \pm \sqrt{1 - \dfrac{\Omega}{\Lambda}}\right); & 4A_{2\gamma} > A_\gamma \\ \dfrac{\Omega}{2}, & 4A_{2\gamma} = A_\gamma \end{cases} \tag{14}$$

$$\Lambda = \frac{A_\gamma\theta' - A_{2\gamma}(\beta_0 + \Theta')}{4A_{2\gamma} - A_\gamma}; \qquad \Omega = \frac{A_\gamma(2 - \theta) - A_{2\gamma}\left(1 - (\beta_0 + \Theta')^2\right) + 2A_0}{A_\gamma\theta' - 2A_{2\gamma}(\beta_0 + \Theta')}.$$

By calculating the angle γ, one can determine the energy utilization coefficient using formulas (2).

4. Results and discussion

From formulas (14), it is evident that the dependence of the angle γ on the aerodynamic characteristics of the blade profile depends on the fulfillment of the following inequality:

$$4\left(B_y + B_x\right)^2 + \frac{32}{\sigma}\left(B_y + B_x\right)\sin\phi + \frac{64}{\sigma^2} \geq A_y^2 + A_x^2, \tag{15}$$

which, taking into account (12), can be rewritten in the form of restrictions on the rotor integrity factor (Table 4) or on the twist angle of its blade cross-section

$$\sin\phi \geq \frac{\left\lfloor A_y^2 + A_x^2 - 4\left(B_y + B_x\right)^2 \right\rfloor \sigma^2 - 64}{32\sigma\left(B_y + B_x\right)},$$

the fulfillment of which ensures or does not ensure inequality (15) and determines the type of dependence in (14).

Such a complex relationship between the parameters of flow interaction with the blade and the aerodynamic characteristics of the profile, the rotor integrity coeficient, and the twist angle of its blade cross-section was established analytically for the first time. It should be understood that the rotor solidity coefficient and the twist angle of its blade cross-section can be variable along the blade length, while the aerodynamic characteristics of the profile remain unchanged. That is, condition (15) may not be fulfilled for all blade sections, i.e., when using (14), different variants of the dependence will be applied for different sections.

Table 4 – Parameters for approximating the aerodynamic characteristics of profiles

Ratio of aerodynamic characteristics of the profile	Condition of execution (15)	Notes
$A_y^2 + A_x^2 > 4\left(B_y + B_x\right)^2$	$\sigma \leq \sigma$	$\sigma = \dfrac{\dfrac{4}{B_y + B_x}}{\sqrt{\dfrac{A_y^2 + A_x^2}{4\left(B_y + B_x\right)^2} - cos^2\phi - sin\phi}}$
$A_y^2 + A_x^2 < 4\left(B_y + B_x\right)^2$	$\sigma \geq \sigma$	

5. Conclusions

Thus, as a result of the improvement of the Sabinin-Yuriev model, an analytical solution to the coupling equation was proposed for the first time, which allows avoiding iterative algorithms when calculating the inductive velocities introduced into the air flow by the wind turbine rotor under given wind flow parameters, rotor rotation conditions, aerodynamic and geometric parameters of the rotor.

So far, the limits of possible use of the proposed solution are conditioned by the conditions of admissibility of the decomposition of the trigonometric functions sine and cosine into a power series while preserving the first two terms, and these limits obviously still need to be further clarified and substantiated. Also, the issue of identifying one real root out of three possible in formula (14) needs to be further resolved, but, in this form, formulas (7)–(14) for the first time allow us to calculate the characteristics of WTR with a horizontal axis of rotation without the use of iterative procedures associated with the numerical solution of the coupling equation, and simplifies the solution of problems of optimizing the parameters of WTR with a horizontal axis of rotation and their maximum adaptation to the conditions of the CIE.

By determining the dependences of the aerodynamic characteristics of more than 50 of the most well-known body profiles on the trigonometric functions of the angle of attack, it is possible to assess the prospects of their use for wind turbine rotors with a horizontal axis of rotation in the conditions of IS of CIE.

Conflict of interest

Authors state no conflict of interest.

REFERENCES

1. Chemeris, I.F., Oksen, Yu.I. and Bokij, B.V. (2006), “Use of the energy potential of the mine ventilating jet”, *Geo-technical mechanics*, vol. 67, pp. 359 – 370.

2. Pivnyak, G.G., Shkrabets, F., Noiberher, N. and Tsyplenkov, D. (2015), *Osnovy vitroenerhetyky* [Basics of wind energy], NMU, Dnepropetrovsk, Ukraine.

3. Drukovanyi, M.F. and Yanovych, V.P. (2016), *Alternatyvni dzherela enerhii* [Alternative energy sources], Vinnytsia National Agrarian University, Vinnytsia, Ukraine.

4. Pivnyak, G.G. and Shkrabets, F.P. (2013), *Alternatyvna enerhetyka v Ukraini* [Alternative energy in Ukraine], National Mining University, Dnepropetrovsk, Ukraine.

5. Tarasov, S.V., Shkrabets, F.P., Zadontsev, V.A. and Otchich, S.V. (2014), *Vetroenergetika. Informatsionno-analiticheskiy obzor po alternativnoy energetike* [Wind Energy. Information and analytical review on alternative energy sector], in Dzenzerskoho, V.A. and Pivnyak, G.G. (ed.), National Mining University, Dnepropetrovsk, Ukraine.

6. Dzenzerskyi, V.A., Tarasov, S.V. and Kostyukov, I.Yu. (2011), *Vetroustanovki maloy moschnosti* [Small wind turbines], Scientific Opinion, Kiev, Ukraine.

7. Abramovskii, Ye.R. and Lychagin, N.N. (2014), *Problemy optimizatsii parametrov venrovykh dvigatelei* [The mathematical modeling and optimal design of wind engine of different capacities and purposes], Science and Education, Dnipropetrovsk, Ukraine.

8. Abramovskii, Ye.R., Avrakhov, F.I., Lychahin, M.M. and Leshchenko, I.H. (2012), *Zadachi i vpravy z vitroenerhetyky* [Problems and exercises in wind energy], Science and Education, Dnepropetrovsk, Ukraine.

9. Abramovskii, Ye.R., Gorodko, S.V. and Sviridov, M.V. (1987), *Aerodinamika vetrodvigateley: ucheb. posobie* [Aerodynamics of wind engines: tutorial. benefit], DNU, Dnepropetrovsk, Ukraine.

10. Chmielniak, T. (2008), *Technologie energetyczne*, Wydawnictwa Naukowa-Techniczne, Warszawa, Poland.

11. Chiras, D. (2010), *Wind Power Basics*, New Society Publishers, Gabriola Island, BC, Canada.

12. Ivanov, O.B., Shkrabets, F.P. and Zawilak, J. (2011), *Electrical generators driven by renewable energy systems*, Wroclaw University of Technology, Wroclaw, Poland.

13. Paska, J. (2005), *Wytwarzanie rozproszone energii elektrycznej i ciepła,* Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa, Poland.

14. Medvedieva, O. O., Larionov, H. I. and Halchenko, Z. C. (2024). "To the selection of technology parameters for the use of renewable energy sources on man-made disturbed lands", IOP Conference Series: Earth and Environmental Science, vol. 1319(1), 012011. https://doi.org/10.1088/1755-1315/1319/1/012011

15. Woofenden, I. (2009), Wind Power for Dummies, Published by Wiley Publishing, Indianapolis, USA.

16. Burton, T., Sharpe, D., Jenkins, N. and Bossanyi, E. (2011), Wind energy. Handbook. Second Edition, John Wiley & Sons, Ltd., Chichester, New York, Weinheim, Brisbane, Singapore, Toronto. https://doi.org/10.1002/9781119992714

17 Hadnagy, I., Tar, K. and Molnar, J. (2020), "Analysis of the current state of wind power in the world, Europe and Ukraine, especially in Transcarpathia", Ukrainian Geographical Journal, vol. (1), pp. 59-70. https://doi.org/10.15407/ugz2020.01.059

18. Sinchyk, O.M., Mykhailychenko, D.A., Boyko, S.M., Horodniy, O.M. (2013), “Features of operation of the autonomous wind-

power installation in the underground mine iron workings or mines", *Visnyk Of Chernihiv State Technological University. Series. Technical Sciences*, no. 3 (67), pp. 224–232.

19. Kudria, S.O. (ed.) (2020), *Atlas enerhetychnoho potentsialu vidnovliuvanykh dzherel enerhii Ukrainy* [Atlas of energy potential of renewable energy sources in Ukraine], The Institute of Renewable energy of the National Academy of Sciences of Ukraine, Kiev, Ukraine.

20. Zabarny, G.M. and Shurchkov, A.V. (2002), *Enerhetychnyi potentsial netradytsiinykh dzherel enerhii Ukrainy* [Energy potential of non-conventional energy sources in Ukraine], Institute of Engineering Thermophysics, Kiev, Ukraine.

21. Kovalko, M.P. and Denysiuk, S.P. (2005), *Elektrozberezhennia – priorytetnyi napriamok derzhavnoi polityky Ukrainy* [Electricity saving is a priority area of Ukraine's state policy], Ukrainian Electrochemical Congress, Kiev, Ukraine.

22. Konokhov, M.M. and Tsykhmistro, S.I. (2011), "Integrated development of wind power and coal industry in Donbass", *Energosberezhenie*, no. 6, pp. 20–21.

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ДОСЛІДЖЕННЯ МОЖЛИВОСТІ РОЗВ'ЯЗАННЯ РІВНЯННЯ ЗВ'ЯЗКУ ДЛЯ МОДЕЛІ САБІНІНА-ЮР'ЄВА В АНАЛІТИЧНОМУ ВИГЛЯДІ

Семененко Є., Медяник В., Буряк С., Хамініч О., Гальченко З.

Анотація. Ефективність перетворення промислових майданчиків (ПМ) підприємств вугільної галузі (ПВГ) в вітроенергетичні станції визначається не тільки обґрунтуванням діаметру ротору, розподілом куту повороту лопаті та довжиною хорди, а й обираючи певний профіль перетину лопаті, аеродинамічні властивості якого надають перевагу в цих умовах. Результати дослідження залежностей аеродинамічних характеристик більш як 50 найбільш відомих тілесних профілі від тригонометричних функцій кута атаки дозволили запропонувати нові апроксимаційні залежності. Ці залежності спрощують оцінку перспективності використання профілів різного типу для роторів вітроенергетичних установок (ВЕУ) з горизонтальною віссю обертання в умовах ПМ ПВГ. Це дозволило розробити аналітичний метод розв'язання рівняння зв'язку в математичній моделі Сабініна-Юр'єва процесу взаємодії повітряного потоку з ротором ВЕУ з горизонтальною віссю обертання. Це рівняння пов'язує між собою параметри вітрового потоку, умов обертання ротора, аеродинамічних та геометричних якостей ротора, та величини індуктивних швидкостей, які вносить в потік повітря ротор ВЕУ з горизонтальною віссю обертання. В результаті такого вдосконалення моделі Сабініна-Юр'єва вперше отримано аналітичне розв'язання рівняння зв'язку, що дозволяє уникнути ітераційних алгоритмів при розрахунках індуктивних швидкостей, які вносить в потік повітря ротор вітроенергетичної установки при заданих параметрах вітрового потоку, умовах обертання ротору, аеродинамічних та геометричних показниках ротору. Формули, що запропоновані, вперше дозволяють створити алгоритми розрахунку характеристик роторів ВЕУ з горизонтальної віссю обертання без застосування ітераційних процедур, що пов'язані з численним розв'язанням рівняння зв'язку, та спрощують вирішення задач оптимізації параметрів роторів ВЕУ з горизонтальною віссю обертання, забезпечуючи тим самим їх максимальне пристосування до умов ПМ ПВГ.

Ключові слова: вітроенергетична установка з горизонтальною віссю обертання, коефіцієнт використання енергії вітру, модель Сабініна-Юр'єва, швидкохідність ротору, кут атаки.